PART I: MATHEMATICS

SECTION – I

Single Correct Choice Type

This section contains 8 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4) for its answer, out of which **ONLY ONE** is correct.

1. If the function $f(x) = {}^{8-x}P_{x-2}$, then its domain and range are:

   (1) {2, 3, 4, 5}, {1, 5, 12} (2) {3, 4, 5}, {1, 2} (3) {1, 2, 3, 4}, R (4) R, R

2. The number of ordered pairs (m, n) m, $n \in \{1, 2, \ldots, 100\}$ such that $7^m + 7^n$ is divisible by 5 is

   (1) 1250 (2) 2000 (3) 2500 (4) 5000

3. If $e^{ix} = \cos x + i\sin x$ and $x + iy = \begin{vmatrix} 1 & e^{\pi i/4} & e^{\pi i/3} \\ e^{-\pi i/4} & 1 & e^{2\pi i/3} \\ e^{-\pi i/3} & e^{-2\pi i/3} & e^{-2\pi i} \end{vmatrix}$ then

   (1) $x = -1, y = \sqrt{2}$ (2) $x = 1, y = \sqrt{2}$ (3) $x = -\sqrt{2}, y = \sqrt{2}$ (4) None of these

4. Consider a function $h(x) = \min(\sin x, \cos x)$.

   The value of integral $\int_0^{\frac{\pi}{2}} h(x)\,dx$ equals

   (1) $\sqrt{2} - 2$ (2) $2 - \sqrt{2}$ (3) $2\sqrt{2}$ (4) $2 + \sqrt{2}$

5. Let $\vec{A} = 2\hat{i} + \hat{j} - 2k$ and $\vec{B} = \hat{i} + \hat{j}$. If $\vec{C}$ is a vector such that $\vec{A}.\vec{C} = |\vec{C}|, |\vec{C} - \vec{A}| = 2^{3/2}$ and the angle between $\vec{A} \times \vec{B}$ and $\vec{C}$ is 30^0, then $|(\vec{A} \times \vec{B}) \times \vec{C}|$ is equal to

   (1) 2/3 (2) 2 (3) 3/2 (4) 1

6. Some 6-digit numbers are formed from the digits 1, 2, 3, 4, 5 such that each number satisfies the following conditions:

   (i) a digit either doesn't occur; or occurs more than once, and

   (ii) all occurrences of a digit are consecutive

   How many such numbers can be formed?

   (1) 65 (2) 85 (3) 125 (4) 145

7. A line L has intercepts 'a' and 'b' on the coordinate axes. When the axes are rotated through a given angle, keeping the origin fixed, the same line has intercepts 'p' and 'q'. Which of the following statements is true?

   (1) $\mathbf{a^2 + b^2 = p^2 + q^2}$ (2) $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{p^2} + \frac{1}{q^2}$ (3) $a^2 + p^2 = b^2 + q^2$ (4) $\frac{1}{a^2} + \frac{1}{p^2} = \frac{1}{b^2} + \frac{1}{q^2}$

8. Let $f(x) = \sum_{n=50}^{99} (1+x)^n$. The coefficient of x^r in the expansion of f(x) is

   (1) ${}^{100}C_r + {}^{50}C_{r+1}$ (2) ${}^{99}C_{r+1} - {}^{50}C_{r+1}$ (3) ${}^{100}C_{r+1} - {}^{50}C_{r+1}$ (4) ${}^{99}C_r - {}^{50}C_{r+1}$

SECTION – II

Multiple Correct Choice Type

This section contains 4 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4) for its answer, out of which ONE **OR MORE** is/are correct.

9. If f (n) + f (1 – n) = 2, then

(1) $f\left(\frac{1}{2001}\right)+f\left(\frac{2}{2001}\right)+\ldots\ldots+f\left(\frac{2000}{2001}\right)=2000$ (2) f(1/2) = f(1)

(3) f(1/2) + f(– 1/2) = 0 (4) $\sum_{r=1}^{500} f\left(\frac{r}{501}\right)=5\sum_{r=1}^{100} f\left(\frac{r}{101}\right)$

10. Let $f(\theta)=\cos^2\theta+\sin^4\theta$. Then for all the values of θ

(1) $\frac{13}{16}\le A\le 1$ (2) $f(\theta)$ is periodic function with period π

(3) $f(\theta)$ is periodic function with period $\pi/2$ (4) $\frac{3}{4}\le f(\theta)\le 1$

11. If $3\int_0^{2h} f(x)\,dx = h\left[af(0)+bf(h)+cf(2h)\right]$ for all polynomials f(x) of degree ≤ 2, and h > 0, then

(1) b = 4 (2) a = c (3) a = b = c (4) a + b + c = 6

12. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are unit vectors satisfying $\vec{a}\times\left(\vec{b}\times\vec{c}\right)=\frac{\vec{b}}{\sqrt{2}}$, then

(1) $\vec{a}$, $\vec{b}$ and $\vec{c}$ are mutually orthogonal vectors (2) $\vec{a}$ and $\vec{b}$ are orthogonal vectors

(3) angle between $\vec{a}$ and $\vec{c}$ is 45^0 (4) angle between $\vec{a}$ and $\vec{b}$ is 45^0

SECTION – III

Comprehension Type

This section contains 2 groups of questions. Each group has 3 multiple choice questions based on a paragraph. Each question has 4 choices (1), (2), (3) and (4) for its answer, out of which **ONLY ONE** is correct.

Paragraph for Question Nos. 13 – 15:

Let $A=\begin{bmatrix}1 & p\\ r & q\end{bmatrix}$ where p, q, r are chosen from the set {– 2, – 1, 0, 1, 2}

13. How many matrices are possible such that $AA^T = 2I$

(1) 0 (2) only 1 (3) 2 (4) more than 2

14. The probability that A is non-singular is

(1) $\frac{21}{125}$ (2) $\frac{104}{125}$ (3) $\frac{21}{25}$ (4) $\frac{23}{25}$

15. If S is set of symmetric matrices that can be formed, the sum of traces of all the matrices in the set S is

(1) 0 (2) 1 (3) 5 (4) 25

Paragraph for Question Nos. 16 – 18:

Consider the sets: $A_1 = \{1\}$, $A_2 = \{3, 5, 7\}$, $A_3 = \{9, 11, 13, 15, 17\}$, $A_4 = \{19, 21, 23, 25, 27, 29, 31\}$ and so on

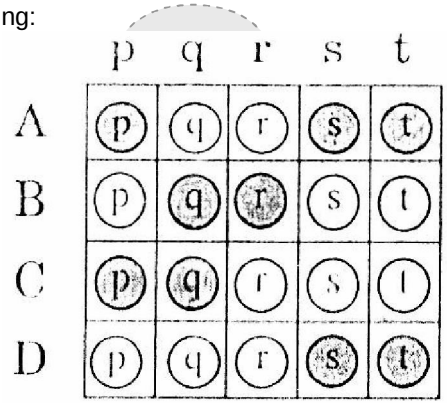
16. What is the least value of n for which the average of elements of A_n is greater than 481?
    (1) 15 (2) 16 (3) 17 (4) 18
17. The sum of elements of A_{10} is
    (1) 2482 (2) 3439 (3) 3349 (4) 4349
18. Which set will contain the element 801?
    (1) A_{20} (2) A_{21} (3) A_{22} (4) None of these

SECTION – IV

Matrix – Match Type

This section contains 2 questions. Each question contains statements given in two columns, which have to be matched. The statements in **Column I** are labeled A, B, C and D, while the statements in **Column II** are labeled p, q, r, s and t. Any given statement in **Column I** can have correct matching with **ONE OR MORE** statement(s) in **Column II**. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following example:

If the correct matches are A – p, s and t; B – q and r; C – p and q; and D – s and t; then the correct darkening of bubbles will look like the following:



19. Consider all possible permutations of the letters of the word **BALLOON**

Column-I	**Column-II**
(A) The number of permutations in which vowels occupy the extreme positions and also no two vowels are together	(P) $9 \times \frac{4!}{2!}$
(B) The number of permutations in which vowels occupy the prime numbered positions.	(Q) $\frac{4!}{2!} x \frac{4!}{2!}$
(C) The number of permutations in which letter A will be preceded and followed by letter O	(R) $7 \times 5!$
(D) The rank of the word **BALLOON** is	(S) $\frac{6!}{2!2!} + 3$

20.

Column-I	Column-II
(A) The locus of the point of intersection of two perpendicular tangents to the parabola $x^2 = 32y$ is a	(P) Circle
(B) The locus of point of intersection of straight lines $\frac{x}{a} - \frac{y}{b} = c$ and $\frac{x}{a} + \frac{y}{b} = \frac{1}{c}$ c $\neq 0$ is a	(Q) Parabola
(C) The locus of complex numbers $z = (k - 1) + i\sqrt{2 - k^2}$ is	(R) Ellipse
(D) The locus of points represented by the parametric equation $x = \alpha^2 + \alpha$, $y = \alpha^2 - \alpha$ is a	(S) Hyperbola
	(T) Line