



IMPORTANT FORMULAE

Class XII - Math
Chapter: Integral Calculus

Concepts and Formulae

S.No	Chapter	Formulae	
1	Integrals	1.1	<p>Indefinite Integrals $\int f(x)dx = F(x) + C$ where $F(x)$ is the antiderivative of $f(x)$</p> <p>Properties</p> $\int [f(x) + g(x)]dx = \int f(x)dx + \int g(x)dx$ $\int kf(x)dx = k \int f(x)dx \text{ for any real number } k$ $\int [k_1f_1(x) + k_2f_2(x) + \dots + k_nf_n(x)]dx = k_1 \int f_1(x)dx + k_2 \int f_2(x)dx + \dots + k_n \int f_n(x)dx$ <p>where, k_1, k_2, \dots, k_n are real numbers & f_1, f_2, \dots, f_n are real functions</p>
		1.2	<p>Some Standard Integrals</p> $\int x^n dx = \frac{x^{n+1}}{n+1} + C, n \neq -1$ $\int dx = x + C$ $\int \cos x dx = \sin x + C$ $\int \sin x dx = -\cos x + C$ $\int \sec^2 x dx = \tan x + C$ $\int \operatorname{cosec}^2 x dx = -\cot x + C$ $\int \sec x \tan x dx = \sec x + C$ $\int \operatorname{cosec} x \cot x dx = -\operatorname{cosec} x + C$ $\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x + C$ $\int \frac{dx}{\sqrt{1-x^2}} = -\cos^{-1} x + C$ $\int \frac{dx}{1+x^2} = \tan^{-1} x + C$ $\int \frac{dx}{1+x^2} = \cot^{-1} x + C$ $\int \frac{dx}{x\sqrt{x^2-1}} = \sec^{-1} x + C$



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			$\int \frac{dx}{x\sqrt{x^2 - 1}} = -\operatorname{cosec}^{-1} x + C$ $\int e^x dx = e^x + C$ $\int a^x dx = \frac{a^x}{\log a} + C$ $\int \frac{1}{x} dx = \log x + C$
		<p>1.3 Integration by Partial Fractions</p> <p>A rational function $\frac{P(x)}{Q(x)} = T(x) + \frac{P_1(x)}{Q(x)}$ if degree of $P(x) >$ degree of $Q(x)$ & $\frac{P_1(x)}{Q(x)}$ can be expressed as sum of partial fractions.</p> $\frac{px + q}{(x - a)(x - b)} = \frac{A}{x - a} + \frac{B}{x - b}, a \neq b$ $\frac{px + q}{(x - a)^2} = \frac{A}{x - a} + \frac{B}{(x - a)^2}$ $\frac{px^2 + qx + r}{(x - a)(x - b)(x - c)} = \frac{A}{x - a} + \frac{B}{x - b} + \frac{C}{x - c}$ $\frac{px^2 + qx + r}{(x - a)^2(x - b)} = \frac{A}{x - a} + \frac{B}{(x - a)^2} + \frac{C}{x - b}$ $\frac{px^2 + qx + r}{(x - a)(x^2 + bx + c)} = \frac{A}{x - a} + \frac{Bx + C}{x^2 + bx + c}$ <p>where $x^2 + bx + c$ cannot be factorised further</p>	
		<p>1.4 Integration by substitution</p> <p>A change in the variable of integration often reduces an integral to one of the fundamental integrals. Some standard integrals are:</p> $\int \tan x dx = \log \sec x + C$ $\int \cot x dx = \log \sin x + C$ $\int \sec x dx = \log \sec x + \tan x + C$ $\int \operatorname{cosec} x dx = \log \operatorname{cosec} x - \cot x + C$ <p>Standard substitutions</p> $\sqrt{x^2 + a^2}$ substitute $x = a \tan \theta$ $\sqrt{x^2 - a^2}$ substitute $x = a \sec \theta$ $\sqrt{a^2 - x^2}$ substitute $x = a \sin \theta$ or $a \cos \theta$	



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		<p>1.5 Integral of some special functions</p> $\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \log \left \frac{x-a}{x+a} \right + C$ $\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \log \left \frac{a+x}{a-x} \right + C$ $\int \frac{dx}{x^2 + a^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + C$ $\int \frac{dx}{\sqrt{x^2 - a^2}} = \log \left x + \sqrt{x^2 - a^2} \right + C$ $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1} \frac{x}{a} + C$ $\int \frac{dx}{\sqrt{x^2 + a^2}} = \log \left x + \sqrt{x^2 + a^2} \right + C$
		<p>1.6 Integration by parts</p> $\int f_1(x) \cdot f_2(x) dx = f_1(x) \int f_2(x) dx - \int \left[\frac{d}{dx} f_1(x) \cdot \int f_2(x) dx \right] dx$ <p>where f_1 & f_2 are functions of x ILATE I- inverse trigonometric L- logarithmic A-algebra T-Trigonometric E- exponential, is used to identify the first function.</p>
		<p>1.7 Some special type of integrals</p> $\int \sqrt{x^2 - a^2} dx = \frac{x}{2} \sqrt{x^2 - a^2} - \frac{a^2}{2} \log \left x + \sqrt{x^2 - a^2} \right + C$ $\int \sqrt{x^2 + a^2} dx = \frac{x}{2} \sqrt{x^2 + a^2} + \frac{a^2}{2} \log \left x + \sqrt{x^2 + a^2} \right + C$ $\int \sqrt{a^2 - x^2} dx = \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} + C$
		<p>1.8 Area function</p> $A(x) = \int_a^x f(x) dx, \text{ if } x \text{ is a point in } [a, b]$



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		<p>1.9</p>	<p>Fundamental Theorem of Integral Calculus First fundamental theorem of integral calculus: If Area function, $A(x) = \int_a^x f(x)dx$ for all $x \geq a$, & f is continuous on $[a, b]$. Then $A'(x) = f(x)$ for all $x \in [a, b]$. Second fundamental theorem of integral calculus: Let f be a continuous function of x in the closed interval $[a, b]$ and let F be another function such that $\frac{d}{dx}F(x) = f(x)$ for all x in domain of f, then $\int_a^b f(x)dx = [F(x) + C]_a^b = F(b) - F(a)$</p>
<p>2</p>	<p>Definite Integrals</p>	<p>2.1</p>	<p>Definite integral as limit of sum $\int_a^b f(x)dx = (b - a) \lim_{n \rightarrow \infty} \frac{1}{n} [f(a) + f(a+h) + \dots + f(a + (n-1)h)]$ where $h = \frac{b-a}{n}$</p>
		<p>2.2</p>	<p>Properties of Definite Integrals $\int_a^b f(x)dx = \int_a^b f(t)dt$ $\int_a^b f(x)dx = - \int_b^a f(x)dx$ In particular, $\int_a^a f(x)dx = 0$ $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$ $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$ $\int_0^a f(x)dx = \int_0^a f(a-x)dx$</p>



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			$\int_0^{2a} f(x)dx = \int_0^a f(x)dx + \int_0^a f(2a-x)dx$ $\int_0^{2a} f(x)dx = 2 \int_0^a f(x)dx, \text{ if } f(2a-x) = f(x)$ $= 0, \text{ if } f(2a-x) = -f(x)$ $\int_{-a}^a f(x)dx = 2 \int_0^a f(x)dx, \text{ if } f(-x) = f(x)$ $= 0, \text{ if } f(-x) = -f(x)$
3	Application of Integrals	3.1	<p>Area of bounded region</p> <p>The area of the region bounded by the curve $y = f(x)$, x-axis and the lines $x = a$ and $x = b$ ($b > a$) is $\text{Area} = \int_a^b ydx = \int_a^b f(x)dx$</p> <ul style="list-style-type: none"> The area of the region enclosed between two curves $y = f(x)$, $y = g(x)$ and the lines $x = a$, $x = b$ is $\text{Area} = \int_a^b [f(x) - g(x)]dx \text{ where, } f(x) > g(x) \text{ in } [a,b]$ If $f(x) \geq g(x)$ in $[a, c]$ and $f(x) \leq g(x)$ in $[c,b]$, $a < c < b$, then $\text{Area} = \int_a^c [f(x) - g(x)]dx + \int_c^b [g(x) - f(x)]dx$