Class XII - Math
Chapter: Probability

Concepts and Formulae

Conditional Probability	Definition	If E and F are two events associated with the same sample space of a random experiment, the conditional probability of the event E given that F has occurred, i.e. P (E|F) is given by $P(E \mid F) = \frac{n(E \cap F)}{n(F)}$
	Properties	E and F be events of a sample space S of an experiment 1) $P(S\|F) = P(F\|F)=1$ 2) For any two events A and B of sample space S if F is another event such that $P(F) \neq 0$, $P((A \cup B) \mid F) = P(A\|F)+P(B\|F)-P((A \cap B)\|F)$ 3) $P(E'\|F) = 1 - P(E\|F)$
Multiplication Theorem on Probability	For two events	Let E and F be two events associated with a sample space S. $P(E \cap F) = P(E)\,P(F\|E) = P(F)\,P(E\|F)$ provided $P(E) \neq 0$ and $P(F) \neq 0$.
	For Three Events	If E, F and G are three events of sample space S, $P(E \cap F \cap G) = P(E)\,P(F\|E)\,P(G\|(E \cap F))$ $= P(E)\,P(F\|E)\,P(G\|EF)$
Independent Events	Definition	▪ Let E and F be two events associated with the same random experiment Two events E and F are said to be independent, if (i) $P(F\|E) = P(F)$ provided $P(E) \neq 0$ and (ii) $P(E\|F) = P(E)$ provided $P(F) \neq 0$ (iii) $P(E \cap F) = P(E) \cdot P(F)$ ▪ If E and F are independent events then so are (i) E' and F (ii) E and F' (iii) E' and F'
Bayes' Theorem	Partition of a sample space	A set of events $E_1, E_2, \ldots, E_n$ is said to represent a partition of the sample space S if (a) $E_i \cap E_j = \phi$, $i \neq j$, $i, j = 1, 2, 3, \ldots, n$ (b) $E_1 \cup E_2 \cup E_n = S$ (c) $P(E_i) > 0$ for all $i = 1, 2, \ldots, n$.

	Theorem of Total probability	Let $\{E_1, E_2,...,E_n\}$ be a partition of the sample space S, and suppose that each of the events E_1, $E_2,..., E_n$ has nonzero probability of occurrence. Let A be any event associated with S, then $P(A) = P(E_1)\,P(A\mid E_1) + P(E_2)\,P(A\mid E_2) + ... + P(E_n)\,P(A\mid E_n)$ $= \sum_{j=1}^{n} P(E_j)\,P(A\mid E_j)$
	Bayes' Theorem	If $E_1, E_2,..., E_n$ are n non-empty events which constitute a partition of sample space S and A is any event of nonzero probability, then $P(Ei\mid A) = \dfrac{P(E_i)P(A\mid E_i)}{\sum_{j=1}^{n} P(E_j)P(A\mid E_j)}$ for any I =1,2,3,…n
Random Variables and its Probability Distributions	Random Variable	A random variable is a real valued function whose domain is the sample space of a random experiment.
	Probability distribution of a random variable	The probability distribution of a random variable X is the system of numbers $\begin{array}{lllll} X & : & x_1 & x_2 & \ldots\ldots & x_n \\ P(X) & : & p_1 & p_2 & \ldots\ldots & p_n \end{array}$ where $p_i > 0, \sum_{i=1}^{n} p_i = 1, i = 1,2,3,\ldots.,n$ The real numbers $x_1, x_2,..., x_n$ are the possible values of the random variable X and p_i $(i = 1,2,..., n)$ is the probability of the random variable X taking the value x_i i.e. $P(X = x_i) = p_i$
	Mean of a random variable	The mean of the random variable X is given by: $\mu = \sum_{i=1}^{n} x_i p_i$ The mean of a random variable X is also called the expectation of X, denoted by E(X). Thus, $E(X) = \mu = \sum_{i=1}^{n} x_i p_i$
	Variance of a random variable	The variance of the random variable X, denoted by Var (X) or σ_x^2 is defined as

		$\sigma_x^2 = \mathrm{Var}(X) = \sum_{i=1}^{n} (x_i - \mu)^2 p(x_i) = E(X - \mu)^2$ $\mathrm{Var}(X) = E(X^2) - [E(X)]^2$
	Standard Deviation	$\sigma_x = \sqrt{\mathrm{Var}(X)} = \sqrt{\sum_{i=1}^{n} (x_i - \mu)^2 p(x_i)}$
Bernoulli Trials and Binomial Distribution	Bernoulli Trials	Trials of a random experiment are called Bernoulli trials, if they satisfy the following conditions : (i) There should be a finite number of trials. (ii) The trials should be independent. (iii) Each trial has exactly two outcomes: success or failure. (iv) The probability of success remains the same in each trial.
	Binomial distribution	For Binomial distribution B (n, p), $P(X = x) = {}^nC_x\, q^{n-x} p^x$, $x = 0, 1, \ldots, n$ $(q = 1 - p)$