MATHEMATICS
Class: XII
Mock Paper 1
Solutions

Section A

1. If $A = \{a, b, c\}$ and $B = \{1, 2, 3\}$ and a function $f: A \to B$ is given by $f = \{(a, 2), (b, 3), (c, 1)\}$

Every element of set A is mapped to the unique element of set B.i.e each element in the set B has a unique pre image in B

$\Rightarrow$ f is a one – one function

Range of f = {1, 2, 3}= B

$\Rightarrow$ f is an onto function

$\therefore$ f is a injective function

[1 Mark]

2. The function y = cosx can be inverted in the intervals where it is both one –one and onto i.e in the intervals $[-2\pi, -\pi]$, $[-\pi, 0]$, $[0, \pi]$, $[\pi, 2\pi]$

[1 Mark]

3. $(A + B)^2 = (A + B)(A + B) = A(A + B) + B(A + B)$
$= A^2 + AB + BA + B^2$ which may or may not be equal to $A^2 + 2AB + B^2$

[Since matrix multiplication is not commutative]

So the expression is not true in general. [1 Mark]

4. Let $A = \begin{bmatrix} 3 & -4 \\ 1 & 1 \end{bmatrix} \Rightarrow A' = \begin{bmatrix} 3 & 1 \\ -4 & 1 \end{bmatrix}$

$A - A' = \begin{bmatrix} 3 & -4 \\ 1 & 1 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ -4 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -5 \\ 5 & 0 \end{bmatrix}$

Transpose of $(A - A') = (A - A')'$

$= \begin{bmatrix} 0 & 5 \\ -5 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & -5 \\ 5 & 0 \end{bmatrix} = -(A - A')$

$\Rightarrow (A - A')$ is a skew symmetric matrix (1 Mark)

5. $A = \begin{pmatrix} 2 & -3 \\ 3 & 4 \end{pmatrix}$ and $A^2 = \begin{pmatrix} -5 & -18 \\ 18 & 7 \end{pmatrix}$

$$A^2 - 6A = \begin{pmatrix} -5 & -18 \\ 18 & 7 \end{pmatrix} - 6\begin{pmatrix} 2 & -3 \\ 3 & 4 \end{pmatrix}$$

$$A^2 - 6A = \begin{pmatrix} -5 & -18 \\ 18 & 7 \end{pmatrix} - \begin{pmatrix} 12 & -18 \\ 18 & 24 \end{pmatrix} = \begin{pmatrix} -17 & 0 \\ 0 & -17 \end{pmatrix}$$

$$A^2 - 6A + 17I = \begin{pmatrix} -17 & 0 \\ 0 & -17 \end{pmatrix} + 17\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} -17 & 0 \\ 0 & -17 \end{pmatrix} + \begin{pmatrix} 17 & 0 \\ 0 & 17 \end{pmatrix}$$

$$A^2 - 6A + 17\,I = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

[1 Mark]

6. $I = \int \frac{1}{\sqrt{9 - 25x^2}} dx$

$= \frac{1}{5} \int \frac{1}{\sqrt{\left(\frac{3}{5}\right)^2 - x^2}} dx$

$= \frac{1}{5} \sin^{-1}\left(\frac{x}{\frac{3}{5}}\right) + c$

$= \frac{1}{5} \sin^{-1}\left(\frac{5x}{3}\right) + c$

[1 Mark]

7. The unit vector in the direction of vector $\vec{a} = \hat{a} = \frac{\vec{a}}{|\vec{a}|}$

$\therefore \hat{a} = \frac{3\hat{i} - 2\hat{j} + 6\hat{k}}{|3\hat{i} - 2\hat{j} + 6\hat{k}|} = \frac{3\hat{i} - 2\hat{j} + 6\hat{k}}{\sqrt{9 + 4 + 36}}$

$= \frac{3\hat{i} - 2\hat{j} + 6\hat{k}}{\sqrt{49}} = \frac{3\hat{i} - 2\hat{j} + 6\hat{k}}{7} = \frac{3}{7}\hat{i} - \frac{2}{7}\hat{j} + \frac{6}{7}\hat{k}$

[1 Mark]

8. P(-1,-2,4) and Q(2,0,-2)

position vector of P = $-1\hat{i}-2\hat{j}+4\hat{k}$

position vector of Q = $2\hat{i}+0\hat{j}-2\hat{k}$

$\overrightarrow{PQ}$ = position vector of Q - position vector of P $=\left(2\hat{i}+0\hat{j}-2\hat{k}\right)-\left(-1\hat{i}-2\hat{j}+4\hat{k}\right)$

$=3\hat{i}+2\hat{j}-6\hat{k}$ ($\frac{1}{2}$ mark)

Magnitude of $\overrightarrow{PQ}=\sqrt{3^2+2^2+(-6)^2}=\sqrt{49}=7$ ($\frac{1}{2}$ mark)

9. Given A(3,-5,1), B (-1, 0 , 8) and C (7 , -10 , -6)

$\therefore$ position vector of A = $3\hat{i}-5\hat{j}+1\hat{k}$

position vector of B = $-1\hat{i}+0\hat{j}+8\hat{k}$

position vector of C = $7\hat{i}-10\hat{j}-6\hat{k}$

$\overrightarrow{AB}$ = position vector of B - position vector of A $=\left(-1\hat{i}+0\hat{j}+8\hat{k}\right)-\left(3\hat{i}-5\hat{j}+1\hat{k}\right)$

$=-4\hat{i}+5\hat{j}+7\hat{k}$

$\overrightarrow{AC}$ = position vector of C -position vector of A $=\left(7\hat{i}-10\hat{j}-6\hat{k}\right)-\left(3\hat{i}-5\hat{j}+1\hat{k}\right)$

$=4\hat{i}-5\hat{j}-7\hat{k}$

$\overrightarrow{AC}=-\overrightarrow{AB}$

$\overrightarrow{AB}$ and $\overrightarrow{AC}$ has same magnitude but opposite directions

$\Rightarrow$ The points A, B and C are collinear [1 Mark]

10. Let $f(x)=\sin^7 x$

$f(-x)=\sin^7(-x)=-\sin^7 x=-f(x)$

So f(x) is an odd function of x

$\therefore \int_{-\pi/2}^{\pi/2}\sin^7 x.dx=0$ [1 Mark]

Section B : Section B comprises of 12 questions of four marks each .

11.(i) (a, b) * (c , d) = (ac , ad + b)
(c , d) *(a, b) = (ca , c b + d)
(ac , ad + b) $\neq$ (ca , c b + d)
So, '*' is not commutative

[1 Mark]

(ii) Let (a,b) (c, d) , (e, f) $\in$ A, Then
((a, b)* (c, d)) * (e,f) = (ac , ad + b) * (e,f) = ((ac) e , (ac) f + (ad+b))
= (ace , acf +ad +b)
(a,b)* ((c,d)* (e,f)) = (a,b) * (ce , cf +d) =
(a (ce) , a (cf+d) +b) = (ace , acf +ad +b)

((a, b)* (c, d)) * (e,f) = (a,b) ((c,d)* (e,f))
Hence, '*' is associative. [1 Mark]

(iii) Let (x , y) $\in$A. Then (x , y) is an identity element, if and only if
(x, y) * (a, b) = (a, b) = (a,b) * (x, y), for every (a,b) $\in$A
Consider (x, y) * (a, b) = (xa , xb + y)
(a,b) * (x, y)= (ax, ay +b)
(xa , xb + y) = (a, b) = (ax, ay +b)
[1 Mark]
ax = x a = a $\Rightarrow$ x = 1
xb + y = b = ay +b $\Rightarrow$ b + y = b = ay +b $\Rightarrow$ y = 0 = ay $\Rightarrow$ y = 0
Therefore , (1, 0) is the identity element
[1 Mark]

OR

*	0	1	2	3	4	5
0	0	1	2	3	4	5
1	1	2	3	4	5	0
2	2	3	4	5	0	1
3	3	4	5	0	1	2
4	4	5	0	1	2	3
5	5	0	1	2	3	4

[2 Marks]
From the table, the second row and second column are the same as the original set.
0*0 = 0 , 1*0 = 0*1 = 1 , 2*0= 0*2 = 2 , 3*0= 0*3 = 3 , 4*0 = 0*4 =4, 0*5 = 5*0 = 5
'0'is the identity element of the operation '*'
[1 Mark]

Now, the element '0' appears in the cell 1*5 = 5*1 =0, 2*4 = 4 *2= 0, 3*3 =0, and 0* 0 = 0
Inverse element of 0 is 0, Inverse element of 1 is 5, Inverse element of 2 is 4, Inverse element of 3 is 3, Inverse element of 4 is 2, Inverse element of 5 is 1.

[1 Mark]

12.Given: $\tan^{-1}\left(\frac{1-x}{1+x}\right) = \frac{1}{2}\tan^{-1}x \ (x>0)$

$$\Rightarrow 2\tan^{-1}\left(\frac{1-x}{1+x}\right) = \tan^{-1}x$$

$$\Rightarrow \tan\left[2\tan^{-1}\left(\frac{1-x}{1+x}\right)\right] = \tan\left[\tan^{-1}x\right]$$ [1 Mark]

$$\Rightarrow \frac{2\tan\left[\tan^{-1}\left(\frac{1-x}{1+x}\right)\right]}{1-\left(\tan\left[\tan^{-1}\left(\frac{1-x}{1+x}\right)\right]\right)^2} = x$$ [1 Mark]

$$\Rightarrow \frac{2\left(\frac{1-x}{1+x}\right)}{1-\left(\frac{1-x}{1+x}\right)^2} = x$$

$$\Rightarrow \frac{2(1-x)(1+x)}{(1+x)^2-(1-x)^2} = x$$ [1 Mark]

$$\Rightarrow \frac{(1-x^2)}{2x} = x$$

$$\Rightarrow 3x^2 = 1$$

$$\Rightarrow x = \pm\frac{1}{\sqrt{3}}$$ [1 Mark]

13. $\lim\limits_{x\to 0^-} f(x) = \lim\limits_{x\to 0^-} \frac{e^{\frac{1}{x}}-1}{e^{\frac{1}{x}}+1} = \frac{0-1}{0+1} = -1$ [1 Mark]

$\lim\limits_{x\to 0^+} f(x) = \lim\limits_{x\to 0^+} \frac{e^{\frac{1}{x}}-1}{e^{\frac{1}{x}}+1} = \lim\limits_{x\to 0^+} \frac{1-\frac{1}{e^{\frac{1}{x}}}}{1+\frac{1}{e^{\frac{1}{x}}}} = \frac{1-0}{1+0} = 1$ [1 Mark]

$\lim\limits_{x\to 0^-} f(x) \neq \lim\limits_{x\to 0^+} f(x)$

So $\lim\limits_{x\to 0} f(x)$ does not exist . [1 Mark]

$\Rightarrow f(x)$ is not continuous at $x = 0$ [1 Mark]

14. y= $\sqrt{a+\sqrt{a+\sqrt{a+x^2}}}$,where a is a constant .

$\Rightarrow y = \left[a+\sqrt{a+\sqrt{a+x^2}}\right]^{\frac{1}{2}}$ [1Mark]

$\frac{dy}{dx} = \frac{1}{2}\left[a+\sqrt{a+\sqrt{a+x^2}}\right]^{-\frac{1}{2}} \frac{d}{dx}\left[a+\sqrt{a+\sqrt{a+x^2}}\right]$ [1Mark]

$\frac{dy}{dx} = \frac{1}{2}\left[a+\sqrt{a+\sqrt{a+x^2}}\right]^{-\frac{1}{2}} \left[\frac{1}{2}\left(a+\sqrt{a+x^2}\right)^{-\frac{1}{2}}\right] \frac{d}{dx}\left(a+\sqrt{a+x^2}\right)$ [1Mark]

$\frac{dy}{dx} = \frac{1}{2}\left[a+\sqrt{a+\sqrt{a+x^2}}\right]^{-\frac{1}{2}} \left[\frac{1}{2}\left(a+\sqrt{a+x^2}\right)^{-\frac{1}{2}} \frac{1}{2}\left(a+x^2\right)^{-\frac{1}{2}}.2x\right]$ [1Mark]

$\frac{dy}{dx} = \frac{1}{4}x\left[\left(a+\sqrt{a+\sqrt{a+x^2}}\right).\left(a+\sqrt{a+x^2}\right).\left(a+x^2\right)\right]^{-\frac{1}{2}}$

15.f(x) = $|\sin x|$ in the interval $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$

Let $h(x) = \sin x, g(x) = |x|$

$\therefore goh(x) = f(x) = |\sin x|$

h(x)=sinx is a continuous function in the interval $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$

$g(x) = |x|$ is a continuous function in the interval $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$

$\therefore goh(x) = |\sin x|$ is also a continuous function in the interval $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ [1 Mark]

h(x)=sinx is a differentiable function in the interval $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$

$g(x) = |x|$ is not a differentiable function in the interval $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$

$\therefore goh(x) = |\sin x|$ is also not a differentiable function in the interval $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ [1 Mark]

$\therefore$ Conditions of Lagrange's theorem are not satisfied [1 Mark]

$\therefore$ Lagrange's theorem is not applicable for the given function [1 Mark]

16. $$\int_0^1 \frac{dx}{\sqrt{1+x}-\sqrt{x}} = \int_0^1 \frac{\left(\sqrt{1+x}+\sqrt{x}\right)dx}{\left(\sqrt{1+x}-\sqrt{x}\right)\left(\sqrt{1+x}+\sqrt{x}\right)}$$

$$= \int_0^1 \frac{\left(\sqrt{1+x}+\sqrt{x}\right)dx}{1+x-x}$$

$$= \int_0^1 \left(\sqrt{1+x}+\sqrt{x}\right)dx$$ [1 Mark]

$$= \left[\frac{(1+x)^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^1 + \left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^1$$ [2 Marks]

$$= \frac{2}{3}\left[2^{\frac{3}{2}} - 1 + 1\right]$$

$$= \frac{2}{3}\left[2^{\frac{3}{2}}\right] = \frac{4\sqrt{2}}{3}$$ [1 Mark]

OR

$$\text{Let} I = \int_0^{\pi/2} \sin 2x \ \tan^{-1}(\sin x)dx$$

$$= \int_0^{\pi/2} 2 \sin x \cos x \tan^{-1}(\sin x)dx$$

$$\text{Let} \sin x = t \Rightarrow \cos x dx = dt$$

$$x = 0 \Rightarrow t = 0, x = \frac{\pi}{2} \Rightarrow t = 1$$

$$\therefore I = \int_0^1 2t \tan^{-1} t\, dt = 2\int_0^1 t \tan^{-1} t\, dt$$ [1 Mark]

Integrating by parts , we have

$$= 2\left[\frac{t^2}{2}\tan^{-1} t\right]_0^1 - 2\int_0^1 \frac{t^2}{2(1+t^2)}dt$$

$$= 2\left[\frac{1}{2}\tan^{-1} 1 - 0\right] - \int_0^1 \frac{t^2}{(1+t^2)}dt$$

$$= 2\left[\frac{1}{2}\times\frac{\pi}{4}\right] - \int_0^1 \frac{t^2+1-1}{(1+t^2)}dt$$ [1 Mark]

$$= \frac{\pi}{4} - \int_0^1 \left(1 - \frac{1}{(1+t^2)}\right)dt$$

$$= \frac{\pi}{4} - [t]_0^1 + [\tan^{-1} t]_0^1$$ [1 Mark]

$$= \frac{\pi}{4} - 1 + \frac{\pi}{4}$$

$$= \frac{\pi}{2} - 1$$ [1 Mark]

17.Let x = $\sin\alpha$ and y = $\sin\beta$, such that $\alpha, \beta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\therefore \sqrt{1-\sin^2\alpha} + \sqrt{1-\sin^2\beta} = a(\sin\alpha - \sin\beta)$

$\therefore \cos\alpha + \cos\beta = a(\sin\alpha - \sin\beta)$

$2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2} = 2a\left(\cos\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}\right)$

$\cos\frac{\alpha-\beta}{2} = a\left(\sin\frac{\alpha-\beta}{2}\right)$ [$1\frac{1}{2}$ Marks]

$\cot\frac{\alpha-\beta}{2} = a$

$\frac{\alpha-\beta}{2} = \cot^{-1}a$

$\alpha - \beta = 2\cot^{-1}a$

$\sin^{-1}x - \sin^{-1}y = 2\cot^{-1}a$ [$1\frac{1}{2}$ Marks]

Differentaiting w.r.t x , we have

$\frac{1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-y^2}} \cdot \frac{dy}{dx} = 0$

$\Rightarrow \frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$ [1 Mark]

18. $(x-1)dy + y\,dx = x(x-1)\,y^{\frac{1}{3}}\,dx$

$\Rightarrow (x-1)\frac{dy}{dx} + y = x(x-1)\,y^{\frac{1}{3}}$

$\Rightarrow \frac{1}{y^{\frac{1}{3}}}\frac{dy}{dx} + \frac{y^{\frac{2}{3}}}{x-1} = x$

Let $y^{2/3} = t \Rightarrow \frac{2}{3}y^{-1/3}\frac{dy}{dx} = \frac{dt}{dx}$

$\Rightarrow \frac{3}{2}\frac{dt}{dx} + \frac{t}{x-1} = x$

$\Rightarrow \frac{dt}{dx} + \frac{2}{3}\frac{t}{x-1} = \frac{2}{3}x$ [1 Mark]

This is a linear differential equation, whose integrating factor is

$IF = e^{\int \frac{2}{3}\left(\frac{1}{x-1}\right)dx} = e^{\frac{2}{3}\log(x-1)} = e^{\log(x-1)^{2/3}} = (x-1)^{\frac{2}{3}}$ [$\frac{1}{2}$ Mark]

$\therefore$ Solution of Differential equation is

$t(x-1)^{2/3} = \int \frac{2}{3}x(x-1)^{2/3}dx + c$ [1 Mark]

$\Rightarrow y^{2/3}\ (x-1)^{2/3} = \int \frac{2}{3}x(x-1)^{2/3}dx + c$

$$\Rightarrow y^{2/3}\ (x-1)^{2/3} = \frac{2}{3}\left[\frac{x(x-1)^{5/3}}{\frac{5}{3}} - \int \frac{(x-1)^{5/3}dx}{\frac{5}{3}}\right] + C$$

$$= \frac{2}{5}x(x-1)^{5/3} - \frac{2}{5}\frac{(x-1)^{8/3}}{\frac{8}{3}} + C$$ [1 Mark]

$$\Rightarrow y^{2/3}\ (x-1)^{2/3} = \frac{2}{5}x(x-1)^{5/3} - \frac{3}{20}(x-1)^{8/3} + C$$

$$\Rightarrow y^{2/3} = \frac{2}{5}x(x-1) - \frac{3}{20}(x-1)^2 + C(x-1)^{-2/3}$$ [$\frac{1}{2}$ Mark]

OR

18. $\sec^2 y\frac{dy}{dx} + 2x\tan y = x^3$

Let $\tan y = t \Rightarrow \sec^2 y\ dy = dt$

$\Rightarrow \frac{dt}{dx} + 2tx = x^3$ [1 Mark]

This is a linear differential equation with integrating factor :

IF= $e^{\int 2xdx} = e^{x^2}$ [$\frac{1}{2}$ Mark]

Solution of the differential equation is given by

$t\ e^{x^2} = \int x^3 e^{x^2}dx + C$ [$\frac{1}{2}$ Mark]

To solve $\int x^3 e^{x^2}dx$

Let $x^2 = z \Rightarrow 2x\ dx = dz$

$\Rightarrow \int x^3 e^{x^2}dx = \frac{1}{2}\int ze^z dz$

$= \frac{1}{2}\left[ze^z - \int e^z dz\right] + C$

$= \frac{1}{2}\left[ze^z - e^z\right] + C$

$= \frac{1}{2}(x^2-1)e^{x^2} + C$ [1 Mark]

$\Rightarrow t = \frac{1}{2}(x^2-1) + Ce^{-x^2}$

$\Rightarrow \tan y = \frac{1}{2}(x^2-1) + Ce^{-x^2}$ [1 Mark]

19. Here, $\vec{a}+\vec{b}+\vec{c}=0$

$\Rightarrow \vec{a}+\vec{b}=-\vec{c}$

$\Rightarrow \left(\vec{a}+\vec{b}\right)^2=\left(-\vec{c}\right)^2$ [$\frac{1}{2}$mark]

$\Rightarrow \vec{a}^2+\vec{b}^2+2\vec{a}.\vec{b}=\vec{c}^2$ [1 mark]

$\Rightarrow |\vec{a}|^2+|\vec{b}|^2+2|\vec{a}||\vec{b}|\cos\theta=|\vec{c}|^2$, [1 mark]

where θ is the angle between $\vec{a}$ and $\vec{b}$

$\Rightarrow (3)^2+(5)^2+2(3)(5)\cos\theta=(7)^2$ (1Mark)

$\Rightarrow 9+25+30\cos\theta=49$

$\Rightarrow \cos\theta=\frac{15}{30}=\frac{1}{2}$

$\Rightarrow \theta=\cos^{-1}\left(\frac{1}{2}\right)=60^o$ [$\frac{1}{2}$Mark]

OR

$\vec{a}=\hat{i}-\lambda\hat{j}+3\hat{k}$ and $\vec{b}=4\hat{i}-5\hat{j}+2\hat{k}$

vectors are perpendicular if $\vec{a}.\vec{b}=0$ [1 mark]

$\vec{a}.\vec{b}=\left(\hat{i}-\lambda\hat{j}+3\hat{k}\right).\left(4\hat{i}-5\hat{j}+2\hat{k}\right)$ [1 mark]

$=1\times4+(-\lambda)\times(-5)+3\times2=4+5\lambda+6=10+5\lambda$ [1 mark]

$\Rightarrow 10+5\lambda=0\Rightarrow\lambda=-2$ [1 mark]

20. The given line is $\frac{x+5}{1}=\frac{y+3}{4}=\frac{z-6}{-9}$

Given point is (2, 4, -1)

The distance of a point whose position vector is $\overrightarrow{a_2}$ from a line whose vector equation is $\vec{r} = \bar{a}$

$$d = \frac{\left|\vec{v}\times(\vec{a_2}-\vec{a_1})\right|}{\left|\vec{v}\right|} \qquad (\tfrac{1}{2}\text{Mark})$$

$$= \frac{\left|\left(\hat{i}+4\hat{j}-9\hat{k}\right)\times\left((2\hat{i}+4\hat{j}-1\hat{k})-(-5\hat{i}-3\hat{j}+6\hat{k})\right)\right|}{\left|\left(\hat{i}+4\hat{j}-9\hat{k}\right)\right|} \qquad (\tfrac{1}{2}\text{Mark})$$

$$= \frac{\left|\left(\hat{i}+4\hat{j}-9\hat{k}\right)\times(7\hat{i}+7\hat{j}-7\hat{k})\right|}{\left|\left(\hat{i}+4\hat{j}-9\hat{k}\right)\right|} \qquad (1\text{Mark})$$

$$\left(\hat{i}+4\hat{j}-9\hat{k}\right)\times(7\hat{i}+7\hat{j}-7\hat{k}) = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 4 & -9\\ 7 & 7 & -7\end{vmatrix} = 35\hat{i}-56\hat{j}-21\hat{k} \qquad (1\text{Mark})$$

$$= \frac{7}{\sqrt{98}}\left|\left(5\hat{i}-8\hat{j}-3\hat{k}\right)\right| = \frac{7}{\sqrt{98}}\sqrt{98} = 7 \text{ units} \qquad (1\text{Mark})$$

21. $S = \{(x,y,z) : x,y,z \in \{1,2,3,4,5,6\}\}$

S contains 6×6×6 = 216 cases [1/2 mark]

Let E : an odd number appears atleast once E' : an odd number appears none of the times

i.e E' : an even number appears all three times [1 mark]

$E' = \{(x,y,z) : x,y,z \in \{2,4,6\}\}$

E' contains 3×3×3 = 27 cases [1 mark]

Now, P(E) = 1 - P(E') [1/2 mark]

$= 1 - \frac{27}{216} = 1 - \frac{1}{8} = \frac{7}{8}$ [1 mark]

22. $|A| = \begin{vmatrix} 1 & \sin\theta & 1 \\ -\sin\theta & 1 & \sin\theta \\ -1 & -\sin\theta & 1 \end{vmatrix}$

$= 1(1+\sin^2\theta) - \sin\theta(-\sin\theta + \sin\theta) + 1(1+\sin^2\theta)$

$= 2(1+\sin^2\theta)$ [1 mark]

$0 \le \sin^2\theta \le 1$ [$\frac{1}{2}$mark]

$1 \le (1+\sin^2\theta) \le 2$

$2(1) \le 2(1+\sin^2\theta) \le 2(2)$

$2 \le 2(1+\sin^2\theta) \le 4$ [$1\frac{1}{2}$Marks]

$2 \le |A| \le 4$

$|A| \in [2,4]$ [1 mark]

So value of |A| is in interval [2,4]

Section C : Section C comprises of 07 questions of six marks each.

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Given A= $\begin{bmatrix} 2 & -3 & 3 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{bmatrix}$

A=IA

$$\begin{bmatrix} 2 & -3 & 3 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

$R_1 \to R_1 + R_2 - R_3$

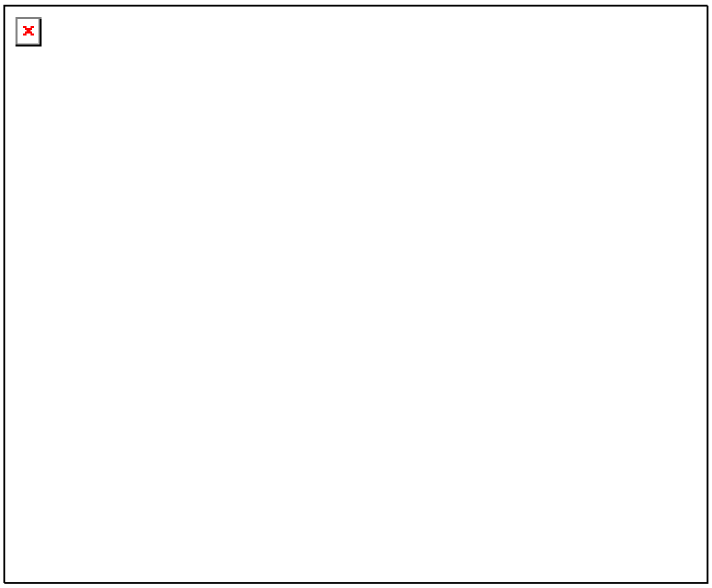
$$\begin{bmatrix} 1 & 1 & 4 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 1 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$ (1 Mark)

$R_2 \to R_2 - 2R_1, R_3 \to R_3 - 3R_1$

$$\begin{bmatrix} 1 & 1 & 4 \\ 0 & 0 & -5 \\ 0 & -5 & -10 \end{bmatrix} = \begin{bmatrix} 1 & 1 & -1 \\ -2 & -1 & 2 \\ -3 & -3 & 4 \end{bmatrix} A$$

$R_2 \leftrightarrow R_3$

$$\begin{bmatrix} 1 & 1 & 4 \\ 0 & -5 & -10 \\ 0 & 0 & -5 \end{bmatrix} = \begin{bmatrix} 1 & 1 & -1 \\ -3 & -3 & 4 \\ -2 & -1 & 2 \end{bmatrix} A$$ [1 Mark]



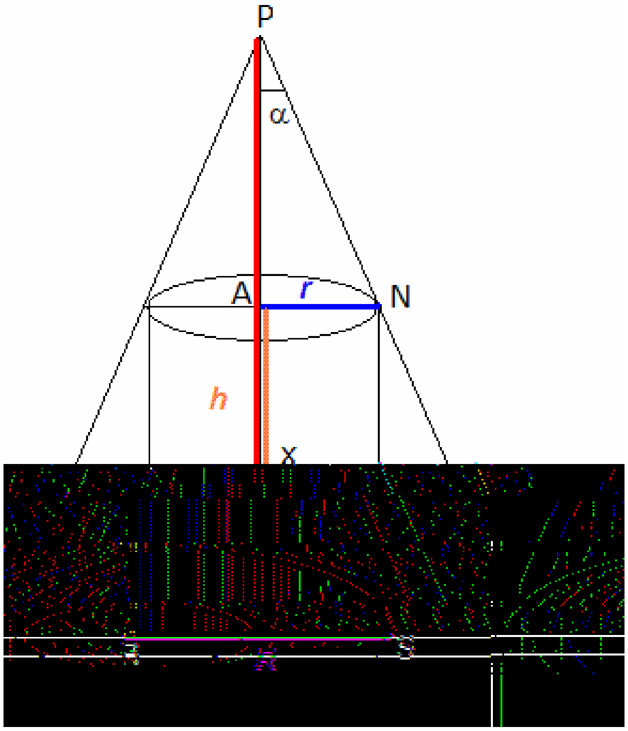
$R_1 \to R_1 - R_2$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -\frac{2}{5} & 0 & \frac{3}{5} \\ \frac{-1}{5} & \frac{1}{5} & 0 \\ \frac{2}{5} & \frac{1}{5} & \frac{-2}{5} \end{bmatrix} A$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = -\frac{1}{5} \begin{bmatrix} 2 & 0 & -3 \\ 1 & -1 & 0 \\ -2 & -1 & 2 \end{bmatrix} A$$ (1 mark)

$$A^{-1} = -\frac{1}{5} \begin{bmatrix} 2 & 0 & -3 \\ 1 & -1 & 0 \\ -2 & -1 & 2 \end{bmatrix}$$ [1 mark]

24.



Let radius of the cone be R and height h. Let r be the radius of the cylinder and x be its height.

Consider triangles PAN and PBS. !PAN ~ !PBS

$$\frac{PA}{PB} = \frac{AN}{BS}$$

$$\Rightarrow \frac{h-x}{h} = \frac{r}{R}$$

$$r = \frac{R(h-x)}{h}$$

The volume of the cylinder

$$V = \pi r^2 x$$

$$= \pi\left[\frac{R(h-x)}{h}\right]^2 x$$

$$= \frac{\pi R^2 (h-x)^2 x}{h^2}$$

$$= \frac{\pi R^2 \left(h^2 x + x^3 - 2hx^2\right)}{h^2}$$ ($1\frac{1}{2}$ Marks)

$$\frac{dV}{dx} = \frac{\pi R^2 \left(h^2 + 3x^2 - 4hx\right)}{h^2}$$

$$\frac{dV}{dx} = 0$$

$$\Rightarrow \frac{\pi R^2 \left(h^2 + 3x^2 - 4hx\right)}{h^2} = 0$$

$$\Rightarrow \left(3x^2 + h^2 - 4hx\right) = 0$$

$$\Rightarrow (3x - h)(x - h) = 0$$

$$\Rightarrow x = h, \frac{h}{3},$$

but $x < h$, so $h = \frac{h}{3}$ [$1\frac{1}{2}$ marks]

$$\frac{d^2V}{dx^2} = \frac{\pi R^2 (6x - 4h)}{h^2}$$ [1 mark]

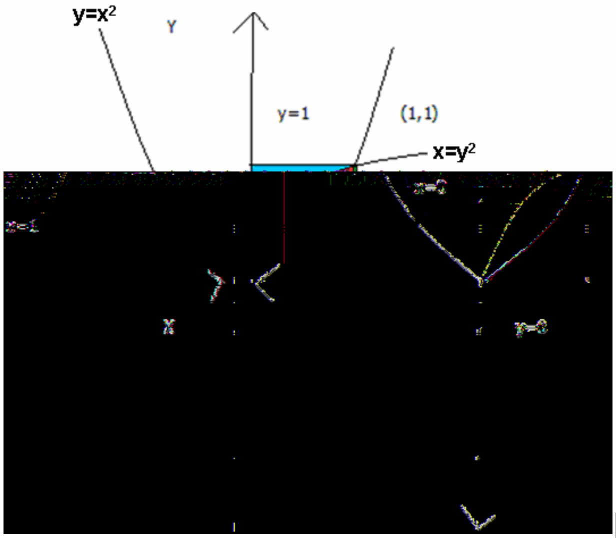
$$\left.\frac{d^2V}{dx^2}\right]_{x=\frac{h}{3}} = \left.\frac{\pi R^2 (6x - 4h)}{h^2}\right]_{x=\frac{h}{3}} = -\frac{2\pi R^2}{h} < 0$$ [1 mark]

x=h/3 is a point of local maxima

$$V\Big]_{x=h/3} = \left.\frac{\pi R^2 (h - x)^2 x}{h^2}\right]_{x=\frac{h}{3}}$$

$$= \frac{4\pi R^2 h}{27} = \frac{4\pi h^3 \tan^2 \alpha}{27}\left[\because \frac{R}{h} = \tan\alpha\right]$$ [1 mark]

25.



[1 mark]

The points where the two parabolas meet in the first quadrant are obtained by solving the two equations $y = x^2$ ....(1) and $x = y^2$. ....(2)

Substituting from (2) into (1) , we get

$X = (x^2)^2$

$\Rightarrow x = x^4$

$\Rightarrow \quad x^4 - x = 0$, i.e $x(x^3 - 1) = 0 \quad \Rightarrow \quad x = 0, 1$

So y = 0,1

$\therefore$The points where the two parabolas meet in the first quadrant are (0,0) and (0, 1).

The area gets divided into 3 parts as shown in three different colours.

[1 Mark]

Area I (In Blue)= $\int_0^1 (1-\sqrt{x})dx = \left[x - \frac{x^{3/2}}{3/2}\right]_0^1 = 1 - \frac{2}{3} = \frac{1}{3}$squnits

Area II (In Red)=$\int_0^1 \left(\sqrt{x} - x^2\right)dx = \left[\frac{x^{3/2}}{3/2} - \frac{x^3}{3}\right]_0^1 = \left(\frac{2}{3} - \frac{1}{3}\right) = \frac{1}{3}$squnits

Area III (In Green)=$\int_0^1 \left(x^2\right)dx = \left[\frac{x^3}{3}\right]_0^1 = \frac{1}{3}$sq.units

Area I =Area II =Area III

$\therefore$The curves $y = x^2$ and $x = y^2$ divide the square bounded by x = 0 , y=0 , x=1 and y=1 into three parts that are equal in area.

[Each area 1 mark , conclusion 1 mark]

26 Let E_1 be the event that a patient used Drug A$\therefore$P(E_1) = ½

Let E_2 be the event that a patient used Drug B$\therefore$P(E_2) = ½

Let E be the event that a patient had a heart attack

Required probability :P(E_1/E)

[1 mark]

$$P(E/E_1) = \frac{40}{100}\left(1 - \frac{30}{100}\right) = \frac{28}{100}$$

$$P(E/E_2) = \frac{40}{100}\left(1 - \frac{25}{100}\right) = \frac{30}{100}$$ [2 marks]

$$P(E_1/E) = \frac{P(E/E_1)P(E_1)}{P(E/E_1)P(E_1) + P(E/E_2)P(E_2)}$$ [1 mark]

$$= \frac{\frac{28}{100} \times \frac{1}{2}}{\frac{28}{100} \times \frac{1}{2} + \frac{30}{100} \times \frac{1}{2}} = \frac{14}{29}$$ [2 marks]

OR

S_1 : the bulb is manufactured by machine X
S_2 : the bulb is manufactured by machine Y
S_3 : the bulb is manufactured by machine Z
Required probability : $P(S_1/E)$
[1 mark]

$P(S_1) = 1/6$
$P(S_2) = 1/3$
$P(S_3) = ½$

$$P(E \mid S_1) = \frac{1}{100}$$

,

$$P(E \mid S_2) = \frac{3}{200}$$

$$P(S_1|E) = \frac{P(S_1)(P(E|S_1))}{P(S_1)(P(E|S_1)) + P(S_2)(P(E|S_2)) + P(S_3)(P(E|S_3))}$$ [1 mark]

$$= \frac{\frac{1}{6} \times \frac{1}{100}}{\frac{1}{6} \times \frac{1}{100} + \frac{1}{3} \times \frac{3}{200} + \frac{1}{2} \times \frac{2}{100}}$$

$$= \frac{\frac{1}{6}}{\frac{1}{6} + \frac{1}{2} + 1}$$

$$= \frac{1}{1+3+6} = \frac{1}{10}$$ [2 marks

27. Given equation of line : $\frac{x-3}{2} = \frac{y-3}{1} = \frac{z}{1}$

Let P be any point on the line , then P(3+2t, 3+t , t)

Now OP makes an angle $\frac{\pi}{3}$, with the given line

$$\cos\frac{\pi}{3} = \frac{|2(3+2t)+1.(3+t)+1.t|}{\sqrt{6}\sqrt{(3+2t)^2+(3+t)^2+t^2}}$$ [1 mark]

$$\frac{1}{2} = \frac{|6t+9|}{\sqrt{6}\sqrt{6t^2+18t+18}}$$ [1 mark]

Squaring and simplifying , we get

$\Rightarrow t^2 + 3t + 2 = 0$

$\Rightarrow (t+1)(t+2) = 0$

[illegible]

28.

$$\frac{x^4}{(x-1)(x^2+1)} = (x+1) + \frac{1}{x^3 - x^2 + x - 1}$$

$$= (x+1) + \frac{1}{(x-1)(x^2+1)} \quad \dots (1)$$

[1 mark]

$$\frac{1}{(x-1)(x^2+1)} = \frac{A}{(x-1)} + \frac{Bx + C}{(x^2+1)} \quad \dots (2)$$

[11/2 marks]

$$1 = A(x^2+1) + (Bx + C)(x-1)$$
$$= (A + B)x^2 + (C - B)x + A - C$$

A + B = 0, C – B = 0 and A – C = 1,

$$A = \frac{1}{2}, B = C = -\frac{1}{2}$$

[1 mark]

$$\frac{1}{(x-1)(x^2+1)} = \frac{1}{2(x-1)} - \frac{1}{2}\frac{x}{(x^2+1)} - \frac{1}{2(x^2+1)} \quad \ldots (3)$$

$$\frac{x^4}{[illegible]} = (x+1) + \frac{1}{[illegible]} - \frac{1}{[illegible]}\frac{x}{[illegible]} - \frac{1}{[illegible]}$$

[11/2 marks]

$$\int \frac{x^4}{(x-1)(x^2+x+1)}dx = \int (x+1)\,dx + \int \frac{1}{2(x-1)}dx - \int \frac{x}{2(x^2+1)}dx - \int \frac{1}{2(x^2+1)}dx$$

[illegible]

[1 mark]

OR

$$\mathrm{I} = \int \left[\sqrt{\cot x} + \sqrt{\tan x}\right] dx = \int \sqrt{\tan x}\,(1+\cot x)\,dx$$

Put $\tan x = t^2$, so that $\sec^2 x\, dx = 2t\, dt$

or $dx = \frac{2t\,dt}{[illegible]}$

[illegible]

[1 Mark]

$$= 2\int \frac{(t^2+1)}{t^4+1}\,dt = 2\int \frac{\left(1+\frac{1}{t^2}\right)dt}{\left(t^2+\frac{1}{t^2}\right)} = 2\int \frac{\left(1+\frac{1}{t^2}\right)dt}{\left(t-\frac{1}{t}\right)^2+2}$$

[1 Mark]

Put $t-\frac{1}{t}=y$, so that $\left(1+\frac{1}{t^2}\right)dt = dy$. Then

[1 Mark]

$$I = 2\int \frac{dy}{y^2+\left(\sqrt{2}\right)^2} = \sqrt{2}\tan^{-1}\frac{y}{\sqrt{2}} + C = \sqrt{2}\tan^{-1}\frac{\left(t-\frac{1}{t}\right)}{\sqrt{2}} + C$$

$= \sqrt{2}\tan^{-1}(t^2-1)$ [illegible] $= \sqrt{2}\tan^{-1}(\tan x - 1)$ [illegible]

[3 Marks]

29.Let the two tailors work for x days

and y days respectively,
The problem is to minimise
the objective function
C= 150 x + 200 y
Subject to the constraints
$6x + 10y \geq 60 \Leftrightarrow 3x + 5y \geq 30$
$4x + 4y \geq 32 \Leftrightarrow x + y \geq 8$
And

$x \geq 0, y \geq 0$

[2 Marks]

Feasible region is shown shaded .

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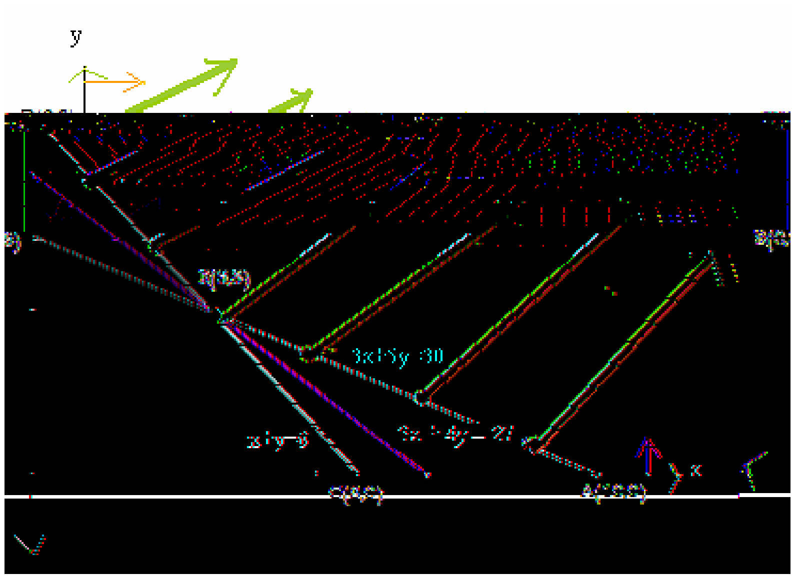

[2 marks]

This region is unbounded

.

corner points	objective functi
A(10,0)	1500
E(5,3)	1350.
D(0,8)	1600

[1 Mark]

The red line in the graph shows the line 150 x + 200y = 1350 or 3x + 4y =27

We see that the region 3x + 4y > 27 has no point in common with the feasible region.

Hence, the function has minimum value at E (5,3).

Hence, The labour cost is the least, when tailor A works for 5 days and Tailor B works for 3 da

[1mark]